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MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LX. No. 6. MARCH 1900.

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E. B. KNOBEL, Esq., PRESIDENT, in the Chair.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Alexander Foote, F.S.A. Scot., Mall Park, Montrose, Scotland ; and 111 Warwick Road, Earl's Court, S.W. (proposed by A. Fowler).

Désiré Ernest Lebon, Agrégé de l'Université, Professeur de mathématiques au Lycée Charlemagne, 4 bis, rue des Écoles, Paris (proposed by C. Flammarion).

One hundred and fifteen presents were announced as having been received since the last meeting, including amongst others :—

Al Battani, Opus Astronomicum, presented by the Milan Observatory; Astronomical and Astrophysical Society of America, First meeting, presented by the Society ; Th. Brédikhine, Sur les radiants composés (dits stationnaires) des étoiles filantes, presented by the author ; Galileo, Opera, vol. 8, presented by the Italian Government ; Sir William and Lady Huggins, Atlas of representative stellar spectra, presented by the author; Rev. S. Kinns, 'Six Hundred Years,' presented by the author ; Königsberg Observatory, Beobachtungen, Abth. 18, 19, presented by the Observatory ; F. X. Kugler, Die Babylonische Mondrechnung, presented by the author ; W. T. Lynn, Remarkable

Comets, 7th edition, and Remarkable Eclipses, 4th edition, presented by the author; L. Weinek, Photographischer Mond-Atlas, Heft 4–6, presented by Professor Weinek; E. T. Whittaker, Report on the progress of the solution of the problem of three bodies, presented by the author; Bronze copy of the Stokes Jubilee Medal, presented by the University of Cambridge.

On the Binary System of Capella. By H. F. Newall.

A very brief announcement of the discovery of the binary character of *Capella* was made in a note communicated to the Society in 1899 November (*ante*, p. 2). A similar announcement was made by Professor Campbell, of the Lick Observatory, in the October number of the *Astrophysical Journal* (vol. x, p. 177). Two brief communications have been made to the Observatory (1900 February, pp. 92, 93). The object of the present note is to lay before the Society the result of a preliminary investigation of the photographs of the spectrum of *Capella* which have been obtained at Cambridge.

A new four-prism spectroscope was attached to the 25-inch equatorial in 1899 July. Some of the earliest photographs obtained with it were spectra of *Capella*; and it was at once noticed that the definition appeared poor and unsatisfactory. From night to night it varied in a curious manner; and it became clear that the peculiarities were real, and not due to instrumental defects, for excellent photographs were obtained of the spectra of other stars—notably of *Procyon* and *Sirius*.

After a preliminary study of ten or twelve photographs of the spectrum of *Capella*, it seemed clear that the spectrum was composite, and lines were picked out as belonging to one component, which we will call the solar component, and other lines as belonging to another component, which in the short range of spectrum dealt with has the characteristics of the spectrum of *Procyon* and *γ Cygni* and *α Persei*; it will be convenient to refer to it in what follows as the *Procyon* component. It is difficult to make out the spectrum; and I think it not unlikely that this choice of name may require revision.

Measurements were then made in the short range of spectrum λλ 4250–4325; and the results in the case of the solar component are given in the following table, and are plotted in the accompanying plate; ordinates representing the velocity of that component relative to the Sun, with time as abscissa. The curve drawn through the observations is a sine curve, to which further reference will be made below. In the table the first column contains the plate number, the second the date, the third the duration of exposure, the fourth the velocity deduced from the photograph (*i.e.* the velocity relative to Earth), the fifth the

correction for orbital motion of the Earth, and the last column the velocity relative to the Sun.

Velocity of Solar Component of α Aurigæ.

	1899.	G.M.T.	Exp.	Vel. rel. to ⊕	Corr. to ⊙	Vel. rel. to ⊙
F. 21	Sept. 28	12.20	60	−23·5	+26·3	+ 2·8
F. 22	Oct. 2	12.19	60	−18·2	+25·7	+ 7·5
F. 32	16	10.58	75	+ 8·3	+22·8	+31·1
F. 33	17	10.30	75	+ 3·2	+22·6	+25·8
F. 34	18	10.54	75	+12·4	+22·3	+34·7
F. 38	Nov. 1	9.55	62	+33·2	+17·9	+51·1
F. 39	6	9.12	75	+34·4	+16·0	+50·4
F. 40	6	10.43	63	+37·3	+16·0	+53·3
F. 44	8	9·56	62	+37·9	+15·2	+53·1
F. 46	10	9.17	70	+41·1	+14·4	+55·5
F. 48	11	10.15	60	+41·8	+14·0	+55·8
F. 49	17	9.16	60	+39·6	+11·4	+51·0
F. 50	20	9.40	60	+39·6	+10·1	+49·7
F. 54	23	9.16	70	+37·8	+ 8·8	+46·6
F. 56	28	8.45	70	+32·3	+ 6·5	+38·8
F. 58	Dec. 1	8.47	70	+27·8	+ 5·1	+32·9
F. 62	2	9.6	60	+28·3	+ 4·6	+32·9
F. 64	11	6.40	50	+22·7	+ 0·3	+23·0
	1900.					
F. 67	Jan. 9	5·57	62			
F. 71	17	12.6	60	+26·6	−16·9	+ 9·5
F. 72	18	12.34	63	+30·1	−17·3	+12·8
F. 73	20	12.22	70	+38·1	−17·8	+20·3
F. 76	24	12.30	70	+42·8	−19·3	+23·5
F. 79	27	11.10	70	+48·4	−20·3	+28·1

The method adopted in deducing the velocity was as follows : The wave-lengths of the chosen lines in the spectrum of the star were determined with reference to the lines in the comparison spectrum of the iron spark, interpolation being performed by means of the relation $\lambda-\lambda_0=c/(R_0-R)$, to which Dr. Hartmann has called attention recently. The three constants, λ_0, c, R_0, are determined from three standard lines in the iron-spark spectrum.

The linear dispersion of the spectra obtained with the instrument is approximately six tenth-metres per millimetre, or 1·5 tenth-metres per revolution of the micrometer used for measuring the spectra. Wave-lengths of lines in the spectra can be determined with reference to the comparison spectra of the iron spark to within about 0·02 of a tenth-metre.

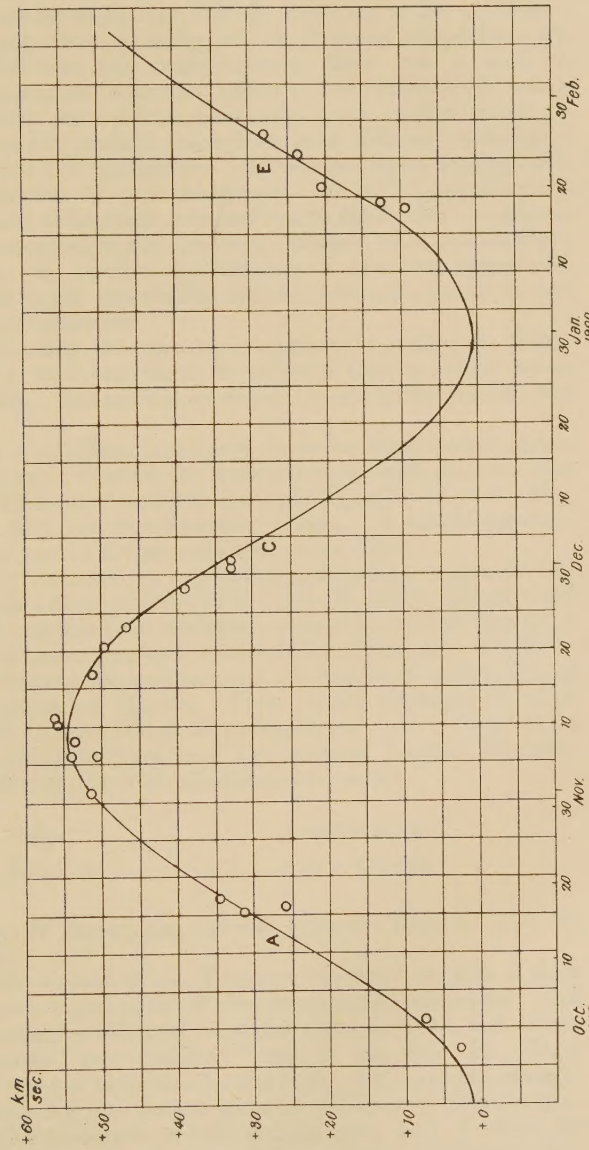
The same three standard lines in the iron-spark spectrum have been used in the reduction of all the plates ; hence any error that may be introduced by assigning to them the wave-lengths given by Rowland for the corresponding lines in the solar spectrum instead of values of wave-lengths in which account is taken of the effects of pressure in the spark, affects all the plates approximately equally.

Thus, in the plate F. 39, taken 1899 November 6, the following wave-lengths were deduced for chosen lines in the star spectrum :

Wave lengths (affected by velocity) of the Star-lines	Wave lengths in Rowland's Tables of Solar Spectrum.	Shift of Line $\delta\lambda$.	Corresponding Velocity.
4275·47	4274·99	+0·48	+33·7 km/sec
76·25	75·68	0·57	40·0
88·77	88·21	0·56	39·1
92·79	92·30	0·49	34·2
93·85	93·28	0·57	39·8
94·77	94·25	0·52	36·3
95·67	95·28	0·39	27·3
4302·13	4301·67*	0·46	32·1
07·07	06·59*	0·48	33·5
14·77	14·34	0·43	30·0
15·59	15·13	+0·46	+32·0
		Mean	+34·4
	Correction for Earth's orbital velocity		+16·0
	Velocity relative to ⊙		+50·4 km/sec.

The velocities come out very consistently considering the difficulties in deciphering the superposed and shifted spectra of the two components. It will be realised that twice in each period the superposed spectra are coincident, line for line ; the lines appear single and distinct, and the spectrum looks in nearly all respects like a well-defined solar spectrum. This occurs when the velocities of the two components are alike, namely, near the points marked A, C, E on Plate 11. At all other times the spectrum looks more or less ill-defined in a curious way ; some lines look double, others are at first sight unexpectedly intensified, others apparently obliterated. Enlarged transparencies of well-defined single spectra have been made (i.) of the Sun, (ii.) of *Procyon* ; when they are superposed on one another, the films being in contact, one spectrum can easily be shifted relatively to the other, and the resultant spectrum has been found to reproduce all the above-described effects in a very striking

* Bright spaces between absorption lines, wave-lengths determined in terms of Rowland from separate photographs.



Velocity of Solar Component of the binary Capella.

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manner. A cursory inspection of the various combinations has been very helpful in guiding one to the choice of lines suitable for measurement. One of the most curious effects brought out is the apparent obliteration of certain groups of lines when the two spectra are appropriately shifted, relatively to one another.

The measurements relating to the *Procyon* component are somewhat difficult, and unfortunately there was a spell of cloudy weather at an important phase of the observations, when the two spectra were shifted relatively to one another in such a way that it would probably have been least difficult to decipher the spectrum of that component. I therefore defer dealing with the motion of the *Procyon* component, and will here only state that it appears to be such as to lead me to think that the masses of the two components are not very different from one another. Furthermore, the relative distinctness of the two superposed spectra points to the probability that the two components do not differ much in brightness.

It is seen that the sine curve appears to satisfy the observations fairly well, but there is evidence that a better curve could be found. The orbit is consequently not quite circular, but nearly so.

A period of 104 days is deduced from the observations of the solar component. Passing the same curve through the velocities deduced by Professor Campbell from photographs taken in 1897, it is easy to pick out corresponding phases. A second approximation to the period is thus obtained, 104·0 days.

Here may be noted an interesting point connected with the Potsdam observations of *Capella* in 1888. It will be remembered that specially careful determinations were made of the velocity of this star, and the concordant results deduced may now clearly be taken as indicating the constancy of the motion of translation of the whole system of *Capella*. From twelve photographs taken between 1888 October 6 and 1889–September 15, Dr. Vogel made twenty-five determinations, and Dr. Scheiner fourteen determinations of the velocity with the definitive results :

Vogel	...	...	+24·8 km/sec.
Scheiner	...	...	+24·1 km/sec.

The velocity of the system at the present time is +27 or +28 km/sec.

It was the custom of the Potsdam observers to give a note of the general appearance of the photographs measured ; and some of the spectra obtained were described as good, and others as "verwaschen" or "unscharf," and in one case Dr. Vogel ascribes the want of definition to the faulty setting of the camera.

In determining the wave-lengths of the lines in *Capella* Professor Scheiner made use of two photographs, each of which he describes as "ganz ausgezeichnetes Spectrum" (*Potsdam Obs.*,

vol. vii., page 257), one taken 1888 October 24 and the other 1888 December 13. It appears that photographs were also obtained on October 22 and on October 25, and that the photograph of October 24 may have been picked out as the best-defined spectrum. Possibly, then, October 24 was an epoch when the velocities of the two components were equal. If so, then the second photograph should correspond with an epoch 52 or 104 days later. The interval between October 24 and December 13 is 50 days, an agreement which may be taken as satisfactory, when it is borne in mind that at the epoch considered the width of the lines in the spectrum increases only at the rate of 0·02 tenth-metre per day.

Again, if the above inference is justifiable and the period 104 days is constant, then the interval between 1888 October 24 and 1899 December 6, which latter date is the epoch in my observations when the velocities were found to be equal and the definition of the spectrum at its best, should be either an exact number of periods or should differ from it by half a period. Now, the interval is 4060 days, or $39{\cdot}04 \times 104$ days. Hence we might arrive at another approximation to the period, viz. 104·1 days; but these are somewhat risky foundations, either for an accurate determination of the period of the system or in support of the view that the period has remained constant.

As to the system of *Capella*, one or two interesting points may be noted.*

The spectroscopic observations show

I. That the components are nearly equal in mass.

II. That the components are not very different in brightness.

III. That the radius of the relative orbit is at least 52,000,000 miles. This is the smallest orbit consistent with the spectroscopic observations; it corresponds to the case when the orbital plane is seen "edge on." Such an orbit would involve eclipses of one component by the other, and consequently also variability in brightness; but no sign of variability has been detected in *Capella*. If the orbit is inclined so that the angle between its normal and the line of vision is i, then the radius of the orbit is $52 \times 10^6/\sin i$ miles.

Let a=actual radius of the relative orbit$= 52 \times 10^6/\sin i$
s=maximum telescopic separation of the components expressed in seconds of arc (not yet observed)
D=distance of the system from the Sun
R=distance of the Earth from the Sun
p=parallax expressed in seconds of arc;

then we have $$\frac{a}{D} = s \,.\, \sin 1''$$

* In this connection reference should be made to an interesting note by Miss Clerke in *The Observatory*, 1900 March, p. 127.

and $$\frac{R}{D} = p \,.\, \sin 1'' ;$$

whence $$a = \frac{sR}{p} = \frac{52 \times 10^6}{\sin i}$$

or $$\frac{s \,.\, \sin i}{p} = \frac{52}{93}.$$

Now, Dr. Elkin gives for the parallax of *Capella* the value $0''{\cdot}08$. And *Capella* has not been observed telescopically as a double-star; its components are therefore presumably less than $0''{\cdot}1$ apart. Hence we infer that to make the spectroscopic observations fit with these data, we must have

$$\sin i > \frac{8 \times 52}{10 \times 93}, \text{ or } {\cdot}447,$$
$$\text{or } i > 27°.$$

Even when $i = 90°$, the maximum telescopic separation of the components, if Elkin's parallax be accepted, must be at least $0''{\cdot}04$.

With respect to the mass of the system, let

M = the mass of the solar component,

nM = the mass of the *Procyon* component,

U = the period of the binary system.

Then we have $$M = \frac{a^3(1+n)^2}{U^2}.$$

Since the solar component moves with an orbital velocity of $\frac{27}{\sin i}$ km/sec we deduce that the radius of its orbit (assumed circular), is

$$a = \frac{38{\cdot}6 \times 10^6}{\sin i} \text{ kilometres or } \frac{24 \times 10^6}{\sin i} \text{ miles.}$$

Hence the mass of the *Capella* system is

$$(1+n)\,M = \frac{24^3}{93^3} \cdot \frac{365^2}{104^2} \cdot \frac{(1+n)^3}{\sin^3 i} \times \text{Sun's mass}$$
$$= 0{\cdot}212 \times \frac{(1+n)^3}{\sin^3 i} \times \text{Sun's mass.}$$

And if, accepting I, we put $n = 1$, we have

$$\text{mass of } \textit{Capella} \text{ system} = \text{Sun's mass} \times \frac{1{\cdot}7}{\sin^3 i};$$

and, accepting Elkin's parallax, and assuming that the components are less than $0''{\cdot}1$ apart, we have

$$\text{mass of } \textit{Capella} \text{ system} < 19 \times \text{Sun's mass.}$$

With reference to the brightness of *Capella*, we have the following considerations:—If the Sun were removed to the distance of *Capella*, it would appear 32 magnitudes fainter than at its present distance. Taking the Sun's magnitude as $-25{\cdot}5$, and *Capella's* as $+0{\cdot}2$, we find

Capella's brightness=480 times the Sun's brightness.

If the components are equal and have the same intrinsic brightness of surface as the Sun, it would appear that each component must have a diameter about 15 times that of the Sun. In this case eclipses could only be avoided if the angle between the normal to the orbit and the line of sight is not greater than that value which satisfies the equation $\frac{15\times 8\times 10^5 \sin i}{52\times 10^6}=\cos i$, or $i=77°$.

These considerations are enough to show that we are nearly within reach of interesting *facts* relating to the evolution of stars. If telescopic observations show that *Capella* is a double star, we shall be in a position to deal with a known case of a star with a spectrum similar to that of the sun, though its mass and brightness may be very different. The complete investigation of the spectrum of the *Procyon* component is also likely to be of great interest in the same connexion.

I have great pleasure in taking this opportunity of saying that I owe much to the skill and care of my assistant, A. W. Goatcher. He has secured more than half of the photographs from which I have made the measurements contained in this note; and I am indebted to him in numberless ways for his unremitting patience and devotion.

Note.—The best chance of detecting doubleness in *Capella* by visual observations through a large telescope will first occur about April 13, or say between April 3 and April 23. The approximate equality in brightness of the components shows that a dark glass may be used at the eye-end.

Photographic Observations of Hind's Variable Nebula in Taurus, made with the Crossley Reflector of the Lick Observatory. By James E. Keeler, D.Sc.

The region of *T Tauri* and Hind's variable nebula has frequently been scrutinised by observers with visual telescopes, but I have been unable to find any record of photographic observations. In making out an observing list for the Crossley Reflector, this region was therefore noted as specially requiring attention.

A complete history of previous observations of this region, together with original observations by himself and Mr. Burnham, has been given by Professor Barnard.* It will therefore be unnecessary for me to review the work of other investigators.

The first photograph with the Crossley Reflector was made on 1899 December 6, with an exposure of $3^h\ 53^m$. It showed a slight darkening of the film near *T Tauri*, which might have been an accidental stain. Accordingly, another photograph, with four hours' exposure, was made on December 27, and it proved to be in exact agreement with the first. Both photographs are good, and the star discs are almost perfectly round, but, for several reasons, they do not show as faint stars, by perhaps several magnitudes, as plates made with equal exposure-times during the fine nights of the previous summer.

Region of *T Tauri* and Hind's Variable Nebula.

From both negatives positives were made on glass with an enlargement of five diameters, and from one of these the accompanying diagram was copied, partly by tracing and partly by measurement. It represents a field 12′ square. The diameters

* *Monthly Notices, R.A.S.*, vol. lv. p. 442.

of the photographic star discs (on the enlargement) are quite approximately reproduced, the largest stars being a little too large, and the smallest a little too small, though the latter are still more conspicuous than they are on the enlargement itself.

Hind's nebula, as shown on the photographs, is faint and very irregular. Three patches, of which the middle one is the brightest, and which are not sharply bounded on any side, are connected by still fainter nebulosity. They do not seem to be connected with *T Tauri*, though it is quite likely that a connection might be traced on a stronger photograph. From the sketches made by Professor Barnard and other observers, it appears that the brightest patch, or the two patches nearly in line with D.M. + 19° 704 are the parts of the nebula which have been observed visually.

The photograph shows no nebulosity whatever at the place of Struve's nebula.

At my request Mr. Aitken and Mr. Perrine examined this region with the 36-inch refractor on January 20 (a fine night). Hind's nebula was seen with difficulty, at the very limit of visibility, as were the faintest stars of the diagram. The magnitude of the latter may therefore lie between 16 and 17. The magnitude of the brighter star (*b*) at the place of Struve's nebula was estimated as 13; that of the smaller star as $15\frac{1}{2}$–16. The latter star was not seen by Professor Barnard, who says particularly that there was no star near *b*. The star is perhaps variable, or the conditions may have been more favourable on the night of Messrs. Aitken and Perrine's examination. Tempel's star near *b* does not agree in position with the star shown by the photograph.

On the night of January 20 both observers estimated that *T Tauri* was about half a magnitude fainter than the star *B* of the diagram. The photographic disc of *T Tauri* is, however, somewhat the larger. The photographic and visual magnitudes of these stars therefore do not agree, unless indeed there was a perceptible variation of the brightness of *T Tauri* between December 27, when the last photograph was taken, and January 20, when the visual observations were made.

Professor Barnard concludes from his investigation that Hind's nebula and Struve's nebula present incontestable cases of nebular variability, and it is likely that all astronomers agree with him. Indeed, it is inconceivable that the nebula near *T Tauri*, if no brighter than it is at present, could have been discovered with a small telescope. With respect to some of the details of the various observations of these nebulæ, which relate to objects at the very limit of vision, there is, I think, room for considerable doubt. So skilful an observer as Tempel has, as I have shown in an article which will be printed in another place,* drawn stars and nebulæ where none exist, while

* *The Astrophysical Journal* for January.

stars and nebulæ that certainly do exist escaped his notice. The fallibility of the observer must not be lost sight of.

Lick Observatory, University of California:
1900 *January.*

On a Simple Method of Comparing the Bonn Durchmusterung with Photographic Plates. By H. H. Turner, M.A., F.R.S., Savilian Professor.

1. The *Bonn Durchmusterung* gives R.A.s and Declinations for 1855·0. Measures of stars on a photographic plate give rectilinear coordinates (x, y) which are connected with standard coordinates (ξ, η) by linear relations

$$\xi = x + ax + by + c$$
$$\eta = y + dx + ey + f,$$

and (ξ, η) are connected with R.A. and Decl. for some other epoch (say 1900·0) by the relations

$$\xi = \tan(\alpha - A)\cos q \sec(q - D) \qquad \eta = \tan(q - D),$$

where $\tan q = \tan\delta \sec(\alpha - A)$,

and (α, δ) are the R.A. and Decl. of the star,

(A, D) „ „ „ „ plate centre.

A circuitous method of comparing the B.D. with the plate is thus to perform the following operations :—

(1) Bring up the B.D. places to 1900·0.

(2) Convert into standard coordinates.

(3) Multiply by the plate constants a, b, c, &c., to get coordinates comparable with measures on the plate.

The present paper indicates a method of dispensing with operations (1) and (3), and generally performing the comparison very quickly.

2. The places of the *Durchmusterung* are given to $0^s{\cdot}1$ in R.A. and $0'{\cdot}1$ in Declination. In terms of a *réseau* interval of $5'$ (a unit now in common use for photographic measures) $0^s{\cdot}1$ on the equator represents 0·005, while $0'{\cdot}1$ represents 0·02. It will probably be sufficient accuracy in both coordinates if we calculate to 0·01 of a *réseau* interval, or in circular measure ·000015.

For the plates of the Astrographic Catalogue

x and y range from $-{\cdot}02$ to $+{\cdot}02$ in circular measure,

x^2 and y^2 „ „ 0 to ·0004,

x^3 and y^3 „ „ $-{\cdot}000008$ to $+ 000008$.

Hence we may neglect powers of the coordinates higher than the *second* for the present purpose; though it is quite easy to take account of these terms afterwards if so desired.

Now to the second order it will be found that

$$\xi=(\alpha-A)\cos\delta, \quad \eta=(\delta-D)+\tfrac{1}{4}(\alpha-A)^2\sin 2D.$$

3. The point of the present method lies in choosing the right value of A. If we increase A by A_0, we add to ξ and η the terms

$$-A_0\cos\delta, \quad -A_0(\alpha-A)\sin D\cos D+\tfrac{1}{4}A_0^2\sin 2D,$$

or if A_0 be of the same order as (ξ and η), since we can put $\cos\delta=\cos D-(\delta-D)\sin D$ to the first order, the terms to be added may be written

$$A_0\eta\sin D+\text{const.} \quad -A_0\xi\sin D+\text{const.}$$

which is equivalent to a rotation of the axes through an angle $A_0\sin D$.

4. It is tolerably obvious, geometrically, that we can get the equivalence of a rotation of axes, or rather of the whole plate, by shifting the centre in R.A.; especially if we visualise the sphere as projected on a plane, say the tangent plane at the Pole. And the significance of the factor sin D is also clear from the geometrical standpoint: the rotation $A_0\sin D$ vanishes at the equator whatever A_0 may be; in other words, the axes of η are all parallel for plates with centres on the equator—we cannot get this rotation there.

5. We proceed to use this principle to get an equivalent for precession. Bringing up the star places from 1855·0 to 1900·0 in the ordinary way is a rather complex numerical process. Viewed as the comparison of two plates of the same stars at different epochs, it merely means that one plate is rotated in its own plane with respect to the other, through a definite angle. To find this angle let us suppose two stars on the axis of η at 1900·0, so that $\xi=0$ for both; and let one be at the centre of the plate, the other Δ south of it, so that $\eta=0$ for the first and $\eta=-\Delta$ for the second. The problem is to choose A, the adopted R.A. of plate centre for 1855·0, so that the value of ξ for the two stars may be, not necessarily zero, but *the same* for both.

We can then reduce both values to zero by subtracting an appropriate constant for the plate, which is a simple matter.

6. If α be the common R.A. of the stars for 1900·0, the precessions for the two stars are

$$m+n\sin\alpha\tan D$$
$$m+n\sin\alpha\tan(D-\Delta).$$

[We are considering differential effects, and the secular variation, even for forty-five years, does not matter.]

Thus the R.A.s for 1855·0 will differ by

$$45n \sin \alpha \, [\tan D - \tan (D-\Delta)].$$

Write this $\beta \sin \Delta \tan D$,

where $\beta = 90n \sin \alpha \operatorname{cosec} 2D$.

Denote the R.A. of the central star for 1855·0 by A_1; that of the southern star by A_2; and that of the appropriate plate centre by A_3. Then we have to find A_3, so that

$$(A_2 - A_3) \cos (D-\Delta) = (A_1 - A_3) \cos D,$$

or

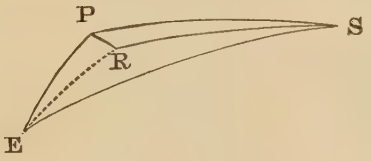
$$\begin{aligned} A_3[\cos D - \cos (D-\Delta)] &= A_1 \cos D - A_2 \cos (D-\Delta) \\ &= A_1[\cos D - \cos (D-\Delta)] \\ &\quad + (A_1 - A_2) \cos (D-\Delta). \end{aligned}$$

Thus $A_3 = A_1 + \beta$

where from above the value of β is

$$\beta = 90n \sin \alpha \operatorname{cosec} 2D.$$

7. It will check this analytical result to look at the question geometrically.



Let E be the pole of the ecliptic,
P be the pole of the equator,
S a star, of coords. (α, D).

Precession carries P to R, in the direction perpendicular to EP, and towards the First Point of *Aries*.

Thus $\angle SPR = \alpha$.

Further $PR = 50''{\cdot}2 \sin \omega \times t$
$= 45n$, as in § 6.

Now PS and RS are the directions of the axis of y for the two plates, which are inclined to one another at

$$\angle PSR = PR \sin \alpha \, . \operatorname{cosec} SP.$$
$$= 45n \, . \sin \alpha \, . \sec D.$$

We wish to correct this by choosing a plate centre differing in R.A. by the quantity β of the last paragraph. And by § 3 or

§ 4 the effect of this change of R.A. is to rotate the plate through an angle $\beta \sin D$.

Hence $\beta \sin D = 45n \sin \alpha \sec D$

or $\beta = 90n \sin \alpha \operatorname{cosec} 2D$.

8. It may be a convenience to others to give here a table of the coefficients of sin α for different declinations, and the rule of signs.

Rule.—To form directly from the *Durchmusterung*, Epoch 1855·0, standard coordinates for 1900·0 : if A_0 be the R.A. for 1855·0 of the actual plate centre, add to A_0 the quantity K sin A_0, where K is given in the following table in seconds of time :—

Dec.	K.	Dec.	K.	Dec.	K.
	min.		s.		s.
0° or 90°	...	15° or 75°	240	30° or 60°	139
1 „ 89	(57)	16 „ 74	226	31 „ 59	136
2 „ 88	(29)	17 „ 73	214	32 „ 58	133
3 „ 87	(19)	18 „ 72	204	33 „ 57	131
4 „ 86	(14)	19 „ 71	194	34 „ 56	130
5 „ 85	(11)	20 „ 70	186	35 „ 55	128
6 „ 84	(10)	21 „ 69	179	36 „ 54	126
7 „ 83	(8)	22 „ 68	173	37 „ 53	125
	sec.				
8 „ 82	435	23 „ 67	167	38 „ 52	124
9 „ 81	388	24 „ 66	162	39 „ 51	123
10 „ 80	350	25 „ 65	157	40 „ 50	122
11 „ 79	320	26 „ 64	152	41 „ 49	121
12 „ 78	295	27 „ 63	149	42 „ 48	121
13 „ 77	274	28 „ 62	145	43 „ 47	120
14 „ 76	256	29 „ 61	141	44 „ 46	120
				45°	120

The method cannot be advantageously employed for plates near the equator or the poles.

Example.—To compare the B.D. with a plate taken with centre at $14^h\ 40^m$, $+37°$ at the Epoch 1900·0.

The R.A. of the plate centre for 1855·0 is $14^h\ 38^m\ 5^s$.

Take from above table the quantity for $+37°$, viz. 125^s. Multiply this by sin $14^h\ 40^m$, which is $-\sin 40°$ or $-·643$, getting -80^s or $-1^m\ 20^s$. Add this algebraically to $14^h\ 38^m\ 5^s$, getting $14^h\ 36^m\ 45^s = A'$ say.

If now (α, δ) be the coordinates of any star for 1855·0, and we form

$$\xi = (\alpha - A') \cos \delta, \quad \eta = (\delta - 37°) + \tfrac{1}{4} (\alpha - A')^2 \sin 74°$$

as in § 2 (where the angles are expressed in circular measure), then ξ and η only differ by a constant from standard coordinates for 1900·0.

9. We have hitherto supposed the plate correctly oriented for 1900·0. Any error of orientation may be corrected in the same way, by again altering the assumed R.A. of plate centre. If (x, y) denote measures on the plate, and (ξ, η) standard coordinates, an error of orientation is indicated by the relations

$$\xi = x - by$$
$$\eta = y + bx,$$

b being (in circular measure) the angle of rotation.

If this is compensated by a change A_3 in the plate centre, we must have by § 3 or § 4

$$\pm b \times 13751 = A_3 \sin D,$$

assuming that A_3 is expressed in seconds of time.

The following table indicates the value of A_3 for different values of b and different centres :—

Table showing how to compensate for the value of b by change of Adopted Plate Centre.

b	Dec.=10° s	20° s	30° s	40° s	50° s	60° s	70° s	80° s	90° s
·001	79	40	28	21	18	16	15	14	14
·002	158	80	55	43	36	32	29	28	28
·003	238	121	82	64	54	48	44	42	41
·004	317	161	110	86	72	64	58	56	55
·005	396	201	138	107	90	80	73	70	69
·006	475	241	165	128	108	95	88	83	83
·007	554	281	193	150	126	111	102	97	97
·008	634	322	220	171	144	127	117	111	110
·009	713	362	248	193	162	143	131	125	124
·010	792	402	275	214	180	159	146	139	138

Rule of Signs.—If b denotes a rotation in the direction above indicated, so that

$$\xi = x - by, \quad \eta = y + bx,$$

(x, y) denoting measures, and (ξ, η) standard coordinates, then the adopted R.A. of the plate centre is to be *increased* by the above quantity before deriving (ξ, η) ; and we shall then get

$$\xi = x + \text{const.}, \quad \eta = y + \text{const.}$$

10. If the plate has a scale value different from that adopted

for (ξ, η), this can be allowed for in a variety of ways. Thus, in forming

$$\xi=(\alpha-\mathrm{A})\cos\delta$$

if we wish to form instead

$$(1+p)\xi=(\alpha-\mathrm{A})(1+p)\cos\delta,$$

we can do so indirectly by writing $(\delta+\epsilon)$ for δ, where

$$\cos(\delta+\epsilon)=(1+p)\cos\delta$$
$$\text{or } \epsilon=-p\cot\delta.$$

It must be remembered that great refinements are not necessary for the comparison of B.D. with our measures, and the mean scale value for a number of plates will probably be near enough for all, in spite of refraction and aberration. At Oxford the focal length of the instrument is nearly one per cent. too great, so that measures made with a standard *réseau* of 5^{mm} spaces have all to be diminished in the ratio 1·0078 to 1. Recently, this troublesome correction (which ought to have been avoided by greater care on the part of the maker of the object-glass) has been obviated by measuring with a *réseau* of $5^{mm}\cdot04$ spaces, so that the scale value is now approximately correct. Suppose then that we have made tables for the more recent plates, as below.

Table for $(\alpha-\mathrm{A})\cos\delta$ *in Réseau intervals.*

Dec′ = 25° 0′	10′	20′	30′	40′	50′	26° 0′
(α − A) m s						
0 10	0·453	0·453	0·452	0·451	0·451	0·450	0·449
20	0·906	0·905	0·904	0·903	0·901	0·900	0·899
30	1·360	1·358	1·356	1·354	1·352	1·350	1·348
40	1·813	1·810	1·853	1·850	1·848	1·845	1·843
50	2·266	2·263	2·259	2·257	2·254	2·250	2·247
1 0	2·719	2·716	2·711	2·708	2·704	2·700	2·696
10	3·172	3·168	3·163	3·159	3·155	3·150	3·146
20	3·626	3·621	3·615	3·610	3·606	3·600	3·595
&c.	&c.	&c.				

the equivalent of the seconds and tenths being found from a supplementary table, and added to the number taken from above before contracting to two figures.

To use this table for comparing the B.D. with measures on

the old plates with defective scale value, we need only substitute for the headings,

$25^\circ\ 0'$, $25^\circ\ 10'$, $25^\circ\ 20'$, $25^\circ\ 30'$, $25^\circ\ 40'$, &c.

the values $25^\circ\ 56'$, $26^\circ\ 6'$, $26^\circ\ 15'$, $26^\circ\ 25'$, $26^\circ\ 34'$, &c.

and the tables are then adapted to the old plates.

For the formation of $\eta=(\delta-\mathrm{D})+\frac{1}{4}(\alpha-\mathrm{A})^2 \sin 2\mathrm{D}$, the second term will not generally need correction, and the first term can be multiplied by the proper factor by means of a small table.

The Use of a Coloured Screen in Photographing the Corona during an Eclipse. By W. Shackleton, A.R.C.Sc.

It has long been known that the distribution of *coronium* throughout the corona was not concurrent with the streamers themselves, the '1474 K' line often being as strong in rifts as elsewhere. From observations made at the eclipse of 1898 both Mr. Newall and Mr. Fowler pointed out that the image of the corona, viewed in the monochromatic light of the green corona line, had decided variations from that seen or photographed directly. In the main the image of the former corresponds to an *inner corona*; this *inner corona* not being merely a sort of under-exposed picture of the whole, but one having a delineation of its own which does not agree with the corona proper. If, then, it were possible to photograph the corona in '1474 K' light alone, we should determine the distribution of *coronium* throughout the corona.

The most obvious way of photographing the corona in the light of the green corona line would be, of course, to use some spectroscopic method; but I wished rather to employ simple apparatus, and from the following considerations I think it possible this may be done by the use of a camera provided with a suitably coloured screen. It will be seen, on examining the photographs taken with prismatic cameras or other spectroscopes during the eclipses of 1893, 1896, and 1898,* that the light of the corona consists principally of continuous spectrum, the line spectrum, with the exception of '1474 K,' being feeble. This line, however, is strong enough to give good decided impressions, although falling in a region of weakness on the isochromatic plates used; indeed, it is so intense that I have seen it recorded on plates supposed to be insensitive to the green.

Referring again to the above photographs, it appears that the continuous spectrum has been photographed to a greater distance from the Sun's limb than has the line spectrum.

* *Phil. Trans.*, A, vol. clxxxvii. (1896), Plates 11, 12; vol. clxxxix. (1897), Plate 9. *Monthly Notices*, R.A.S., vol. lviii., Appendix, Plate 9.

A photograph of the corona on an isochromatic plate sensitive to the rays from yellow to violet, then, is built up of light represented by the continuous spectrum between λ 5900 and λ 3800, together with the light of the line spectrum, represented chiefly by the lines λ 5303 ('1474 K'), λ 4231, λ 3987, the two latter being weak in comparison with the former.

Supposing, therefore, by the use of an absorbing medium we could limit the spectrum to within a short distance on either side of '1474 K,' say by the use of the coloured screen described below, something like 90 to 95 per cent. of the continuous spectrum would be cut off, whilst only a small percentage of line giving light would be absorbed.

Hence by taking a photograph through such a screen we should have the two images, one from the continuous spectrum between λ 5320 and λ 5200, the other from '1474 K' light superposed.

But from the fact that the '1474 K' line is so very intense, and also that the image in this light is of less extent than the ordinary coronal image, it appears very probable that the corona thus photographed would be chiefly due to the *coronium* radiations; the image from the continuous spectrum being relatively weak and spread over a large area, would consequently be too feeble to register in an eclipse of moderately short duration.

From experiments I have made I found it difficult to procure a simple green, but by using two screens stained with the aniline dyes, methylene blue and tartrazine respectively, a resulting colour is produced which allows a narrow band of green light to filter through, the yellow, blue, and violet rays being suppressed. The position of the green line from the corona falls within this band.

The following table shows where the absorption takes place in each case :—

Absorbing Medium.	Edge of Absorption Band. λλ	Rays Absorbed.
Tartrazine—		
(1) Light stain	5100	Green, and more refrangible.
(2) Deep ,,	5290	,, ,, ,,
Methylene blue—		
Medium stain	5320	Yellow, and less refrangible.

It is necessary to use a yellow screen of moderate depth of tint to insure the blue light being well cut off, therefore we may say that the band of green light which is unabsorbed extends from λ 5200 to λ 5320. This being the case, only about 6 per cent. of the light-giving continuous spectrum could pass through, whilst practically all the light of the green corona line would be unopposed.

No doubt some of the green commercial screens used in three-colour photography would be found suitable, but so far I have not tested any of these.

I am indebted to Dr. Morgan, of the Royal College of Science, for samples of various dyes.

Note.—Since the above was communicated to the Society Professor Langley, in a letter to *Nature*,* describes the great intensity of the inner corona, and how in consequence of this brilliance he was able to see *Mercury* in transit projected on the bright background, before it reached the Sun's limb. During the eclipse of 1878 he made a brief telescopic examination of the inner corona, and found it to be "a surprisingly definite filamentary structure . . . . not disposed radially, or only so in the rudest sense, sharpest and much the brightest close to the disc, fading rapidly away into invisibility at a distance of 5′ or more (possibly in some cases ten)."

This description of the inner corona agrees well, in respect to extension, brightness, and non-coincidence of the ordinary radial streamers, with the image of the corona seen in the light of the green coronal line.

Professor Langley further says "that while most interesting photographs of the inner coronal structure have recently been made, yet that this feature has not yet been done justice to even in the best of them I have seen, and that it perhaps cannot be, with our present means."

It would be interesting to see if by the use of a coloured screen this structural detail could be brought out more distinctly.

In a lecture delivered at the Society of Arts, 1900 March 12, on the "Photography of Colour," Mr. E. Sanger Shepherd states that he rejects liquid solutions and stained glass as unsuitable for colour filters, and pronounces in favour of aniline dyes sealed up in gelatine or collodin, between glasses; by experiment a number of these dyes have been found to be of fair permanency.

The Maximum Duration Possible for a Total Solar Eclipse. By C. T. Whitmell, M.A., President of the Leeds Astronomical Society.

Total solar eclipses are among the grandest of natural phenomena. Prolonged totality is very uncommon, and its rarity invests it with exceptional interest.

The eclipse of Thales on 585 B.C. May 28, and that visible in Scotland on 1433 A.D. June 17, were remarkable for prolonged totality.

To come to more recent times, the eclipse of 1868 August 17 is said to have exceeded both those just referred to.

By the kindness of Dr. Downing an estimate of the duration of totality of this 1868 eclipse has been made, and—for local

* Vol. lxi. p. 443, 1900 March 8.

noon—the result is $6^m\ 49^s$. I have obtained a similar result. Governor Hennessy observed this eclipse from Barram Point, Borneo, where, in the afternoon, totality lasted $6^m\ 13^s$; and this is, I believe, the longest duration as yet actually observed.

Noon totality occurred in the Gulf of Siam, at E. longitude 102° 38′ and N. latitude 10° 27′. The conditions were very favourable, the Sun being not very far from apogee, the Moon near a favourable perigee, and almost in the zenith; whilst the observer, being in a low latitude, was situated on a rapidly moving part of the Earth.

The corresponding eclipse of the preceding *Saros* occurred on 1850 August 7. Mr. Paine (see *Monthly Notices, R.A.S.*, vol. xxix.) estimates the noon totality at $6^m\ 51^s$. It would be well to check this result, using the present eclipse values of the diameters of the Sun and Moon.

The eclipse of 1886 August 28 in the *Saros*, subsequent to 1868, had a totality of $6^m\ 35^s$, according to the *Nautical Almanac*. For the eclipse on 1901 May 17, the same authority gives a totality of $6^m\ 30^s$ for a place at which the occurrence of totality will be almost at local noon.

The two series of eclipses, 1850, 1868, 1886, &c, and 1883, 1901, 1919, &c., are amongst the most favourable in the whole history of astronomy.

Mr. Crommelin has kindly furnished me with the following results, calculated by him from Oppolzer's data :—

Date.	Duration at Noon.	Position of Noon Point. Longitude.	Latitude.
	m s	°	°
1901 May 18	6 41·6	97 E.	2 S.
1919 May 29	7 6·9	18 W.	4 N.
1937 June 8	7 19·9	131 W.	10 N.
1955 June 20	7 24·5	117 E.	15 N.
1973 June 30	7 19·6	6 E.	19 N.
1991 July 11	7 10·7	105 W.	22 N.

The 1955 noon eclipse will occur not far from Manila, and its totality is the longest for many centuries.

Mr. Crommelin states that the above durations are trustworthy only to the nearest second, and that Oppolzer's data yield durations about eleven seconds in excess of those of the *Nautical Almanac*, which (I may add) are themselves probably some three seconds in excess of the results of observation.

I now proceed to consider the conditions necessary for the longest totality.

For simplicity, suppose at first that the Moon and the Sun move in the plane of the Earth's equator. When the centres of the three bodies are in a line, the Moon being between the Sun and the Earth, there will be a central solar eclipse at local noon. Suppose this to be total. Bring the Moon nearer to the Earth, and the size of her shadow, or umbra, will increase. Her

velocity also will increase. But the former factor far more than compensates for the latter, so that totality is lengthened by bringing the Moon nearer.

This may seem obvious, but I find that, if the Moon moved in a *circular* orbit, the increased velocity, when her distance was less than 104,000 miles, would more than compensate for the larger umbra, and the duration of totality would therefore fall off, if the Moon, moving in a circular orbit, had a radius vector of less than 104,000 miles.

Now take the Sun further away. The umbra again becomes larger, and the Sun's velocity diminishes. This latter result is a drawback, but is far more than compensated for by the increased size of the umbra. Thus, for a prolonged totality, the Moon should be as near, and the Sun as far off, as possible. The duration of totality increases with the size of the umbra, but diminishes with its velocity over the observer. Now, in the case supposed, the velocities of the Sun, the Moon, and the observer on the equator, are all in the same direction, *i.e.* from west to east, and, at local noon, are all perpendicular to the plane of the observer's meridian.

Thus the linear velocity of the umbra over the observer will be equal to that of the Moon, less that of the Sun in relation to the Moon, less that of the observer. We see, then, that so far as velocity is concerned it is better to have the Sun near and the Moon far away ; but, as already explained, the increased size of the umbra (due to the nearness of the Moon and to the distance of the Sun) far more than compensates for its increased velocity.

Take the Earth's equatorial radius as 3963 miles, then, at mean distance, the linear sidereal velocity of the Moon is about 2288 miles an hour ; but her linear synodical velocity in relation to the Sun is only 2117 miles per hour. The observer's velocity on the equator is 1037·5 miles per hour. Thus we get 1079·5 miles per hour as the noon velocity of the umbra over the observer. During the brief duration of even the longest totality the directions and values of these velocities will remain practically unchanged.

As a matter of fact, for mean distances of the Sun and Moon there would be no total eclipse, but the velocity values given above may be of interest.

There are really five conditions required for maximum totality.

1. The new Moon, at or very near a node, must be at the most favourable perigee possible.

2. The Sun must be at apogee.

3. During totality, which should be observed at local noon, the Moon's shadow must run along a parallel of latitude in order that the diurnal movement of the observer may be for the time parallel to the motion of the Moon, and thus produce its full effect in detaining him within the umbra.

4. The Sun and Moon must be in the zenith, so that the umbra may be as large as possible.

5. The observer must be on the equator, so that his velocity may be as great as possible.

The foregoing conditions could all be simultaneously fulfilled if the Sun and Moon moved in the plane of the equator; but, as this is not the case, we have to make a compromise consistent with the actual facts of nature.

We must keep conditions (1) and (2). Now it is a curious and fortunate thing that condition (3) can then also be fulfilled. The Moon's orbit makes an angle of about 5° with the ecliptic, and this is also about the value of the angle made by the descending ecliptic with that parallel of latitude which cuts it at its apogee point. The Moon must, of course, be in her ascending node, or very near it.

In other words, the Moon's declination, under the given conditions, will be at its monthly maximum, and so will scarcely vary during totality.

It is obvious, however, that conditions (4) and (5) cannot be satisfied simultaneously. Condition (4) puts the observer near the Tropic of Cancer. This increases the umbra, but lessens the observer's velocity. Condition (5) puts him on the equator. This increases his velocity, but diminishes the umbra, for the Moon is no longer in the zenith. We shall find that neither of these positions is the best possible, though (5) is more favourable than (4).

Suppose the Sun at apogee; what perigee distance can the Moon then have, consistently with her being new and at her node? For valuable help in determining this point I am indebted to Mr. Cowell, of Greenwich Observatory; but he is in no way responsible for my numerical results. Using Delaunay's "Lunar Theory," I compute that the Moon's horizontal parallax, under the above conditions, amounts to 61′ 22″. I shall be very glad to have this result confirmed. Under other conditions, such as the Moon being full, or 90° from her node, or the Sun being at perigee, the Moon's parallax may be greater, but these conditions are inconsistent with the problem under consideration.

Suppose the Earth to be a sphere, radius 3963 miles. Put the Sun at apogee, in N. declination 22° 53′ 55″, with an angular semi-diameter of 15′ 43″·78, this being calculated from the reduced mean distance semi-diameter, 15′ 59″·63, used for eclipses. Let the new Moon be at perigee, and at her node, with a parallax of 61′ 22″. The linear semi-diameter of the Moon is about 1081·5 miles, and her geocentric angular semi-diameter is 16′ 44″·77, if we use 15′ 34″ for the semi-diameter at mean distance, and not the smaller "eclipse" value.

Let the observer's geocentric latitude (ϕ) be 22° 53′ 55″ N., and let the Moon be at her node and in the zenith at local noon. The observer's velocity will be less than at the equator, being only 955·74 miles an hour. The Moon's synodic velocity is 2310·50 miles per hour. The diameter of the umbra is 167·89 miles, and its velocity over the observer will be 1354·76 miles

per hour. Putting T for the duration of totality in hours, we obtain for the duration at noon,

$$\text{N. latitude } \phi = 22^\circ\ 53'\ 55'',\ T = \frac{167{\cdot}89}{1354{\cdot}76} = 7^m\ 26^s{\cdot}13 \quad . \quad A$$

In the case just considered the Moon was in the zenith. Now suppose her to have a small geocentric south latitude, so that her shadow falls on the equator. The umbra will be smaller, as the Moon is no longer in the zenith, but the observer's velocity will be greater, so that the velocity of the umbra in relation to the observer is diminished also :

$$\text{Latitude } \phi = 0^\circ\ 0'\ 0'',\ T = \frac{165{\cdot}08}{1273{\cdot}00} = 7^m\ 46^s{\cdot}84 \quad . \quad . \quad B$$

distinctly exceeding (A).

Will this be the maximum totality? To determine this I formed an equation for T, and in this equation the latitude (ϕ) of the observer was practically the only independent variable. Differentiating this, and equating to zero in the usual way, T should be a maximum when $\phi = 4^\circ\ 47'\ 13''$ N. Using this value, we get

$$\text{N. latitude, } \phi = 4^\circ\ 47'\ 13'',\ T = \frac{166{\cdot}12}{1276{\cdot}62} = 7^m\ 48^s{\cdot}45 \quad . \quad C$$

This exceeds (B) by $1^s{\cdot}61$, and is really the maximum under the given conditions. T diminishes for latitudes either N. or S. of this value of ϕ. About N. latitude 9° or 10° T again becomes equal to its value for the equator.

We notice in (B), (C), (A) that the breadth of the umbra and its velocity are both increasing, but at different rates, and that their ratio becomes a maximum in case (C).

But the equation for the latitude has two roots, and the second value of ϕ is $192^\circ\ 32'\ 47''$. This means that we must put the observer in S. latitude $12^\circ\ 32'\ 47''$, and in longitude 180° away from his former position. We must also suppose the Earth transparent, so that the shadow can pass through to the side of it away from the Sun. Granting these suppositions, there will be a central total solar eclipse at local midnight.

$$\text{S. latitude, } \phi = 12^\circ\ 32'\ 47'',\ T = \frac{95{\cdot}97}{3323{\cdot}23} = 1^m\ 43^s{\cdot}96 \quad . \quad D$$

This corresponds to *minimum* duration under the given conditions. The umbra, owing to the observer's increased distance, is much smaller, and its velocity is much greater, for now the observer's velocity has to be added to the synodical velocity of the Moon, for the observer is now moving in a direction opposite to that of the shadow.

This result is only of theoretical interest, but it well illustrates the generality of a mathematical formula, which disdains to take into account such a trifle as the Earth's opacity.

In an actual eclipse of the midnight Sun, the duration of totality is similarly shortened by the increased distance of the observer from the Moon, and by the condition that his velocity has to be added to that of the umbra.

To the foregoing results the following objections may be raised: that the Moon's radius has been taken too large, that the Earth's radius varies with the latitude, and that 3963·296 miles is a more accurate value of the equatorial radius. I have therefore re-calculated the results, using for the Moon's angular semi-diameter at mean distance the "eclipse" value 15′ 32″·65, given in the *Nautical Almanac*. This corresponds to a radius of about 1080 miles, if we take 3963·296 miles for the Earth's equatorial radius, and reckon the Moon's horizontal parallax at mean distance to be 57′ 2″·70. I have also allowed for the variation of the Earth's radius with the observer's latitude.

The durations are now as follows:—

Equator, $\phi = 0°\ 0'\ 0''$, $T = \frac{161{\cdot}93}{1272{\cdot}99} = 7^m\ 37^s{\cdot}94$. E

N. latitude, $\phi = 4°\ 0'\ 0''$, $T = \frac{162{\cdot}82}{1275{\cdot}53} = 7^m\ 39^s{\cdot}54$. F

N. latitude, $\phi = 4°\ 51'\ 45''$, $T = \frac{162{\cdot}99}{1276{\cdot}75} = 7^m\ 39^s{\cdot}58$. G

N. latitude, $\phi = 6°\ 0'\ 0''$, $T = \frac{163{\cdot}20}{1278{\cdot}71} = 7^m\ 39^s{\cdot}46$. H

The latitude for the maximum eclipse (G) is now a little (4′ 32″) higher than before (C), and the duration is about 9 seconds less, a reduction due almost entirely to the Moon's smaller diameter of 2160 miles. But it will be noticed that the excess of duration of (G) over (E) is 1ˢ·64, about the same as it was in the case of (C) and (B). Of course, I assume not complete, but only relative, accuracy for the figures given. But I hope that, with the data used, there is no serious error in the results. For parallax 61′22″ the Moon's semi-diameter is 16′43″·30.

The conclusion, then, is that, with the accepted present eclipse values of the diameters of the Sun and Moon, and with a lunar parallax of 61′ 22″, the maximum eclipse totality will occur near the beginning of July, at noon, in geocentric N. latitude about 4° 52′, and will last about 7ᵐ 40ˢ, the Sun being at apogee with a parallax of 8″·70. The Sun's and Moon's declinations are considered to be practically constant during totality.

If it were possible for the Sun and Moon to be in the plane of the equator, so that the Moon could be in the zenith, the eclipse would last 7ᵐ 45ˢ·88, and the maximum would then be really on the equator. This, however, is impossible.

Mr. G. F. Chambers, in his comprehensive *Handbook of Astronomy* (vol. i. p. 268) quotes from Du Séjour a statement to the effect that maximum totality occurs on the equator, and lasts 7ᵐ 58ˢ. This statement has been copied into numerous text-

books. I have consulted the original paper in *Mémoires de l'Académie Royale des Sciences*, Paris, 1777, p. 318. Du Séjour states that the longest totality will occur at noon on the equator on July 2, with the Sun at apogee. The following data are used by him. Sun's semi-diameter, 15′ 42″; hourly motion in the ecliptic, 2′ 23″; declination, 22° 50′ N. The Moon is near her ascending node, in S. latitude 23′ 57″; hourly motion, reduced to the ecliptic, 38′ 16″; greatest polar parallax, 61′ 17″.

He does not give the Moon's semi-diameter, nor the relation between the polar and the equatorial radius of the Earth. With the ratio at present accepted, a polar parallax of 61′ 17″ would yield an equatorial one of 61′ 29″. He does not refer to the conditions which (I presume) were imposed upon the Moon's parallax by the lunar theory of his time, nor does he appear to be aware that the maximum totality is not actually on the equator. Taking Du Séjour's elements, and assuming for the Moon an equatorial parallax of 61′ 29″, and using her smaller semi-diameter, I get a result practically agreeing with his for an eclipse on the equator. But it seems to me that the parallax is larger than the lunar theory will permit under the conditions, and also that the semi-diameter of the Sun is too small.

Lord Grimthorpe, in his *Astronomy without Mathematics*, writes (p. 151) that "under a combination of the most favourable conditions totality may last something more than 7 minutes." But he bases his calculation upon an umbra only 148 miles in diameter, and this is decidedly under the mark. He also assumes the perigee distance of the Moon to be 221,600 miles, and this corresponds to a parallax of about 61′ 29″. If my computations are correct, this parallax, like that used by Du Séjour, exceeds the value allowed by the lunar theory under the given conditions. In addition to contributing interesting data as to future eclipses, Mr. Crommelin has been good enough to read over the MS. of this paper.

Leeds: 1900 *February.*

Observations of Saturn made at Juvisy Observatory in 1899.
By C. Flammarion.

Observations of *Saturn* were commenced here during the last apparition on 1899 June 1, and continued till July 30. The number of fine nights was abnormally great, a circumstance which more than compensated for the low altitude of the planet above the southern horizon.

The instrument employed was the Juvisy 10½-inch equatorial, bearing powers of 224, 308, 411 and 617, and the observations were made by M. Antoniadi and myself.

1. *The Globe.*

The N. polar cap was not particularly dark in 1899, appearing certainly lighter than it did in 1895. No trace of the N. temperate band ; but the double N. tropical belt was an obvious feature under almost every kind of definition.

The dark spots which sprinkle this belt, and especially its S. component, were missed here in 1899, though vague traces of some seem to have been caught on July 7 and 27.

Also, the equatorial zone did not show its ordinary "wool pack" structure, while the narrow equatorial belt was invisible.

An interesting "black drop" appearance, due to irradiation, was repeatedly detected in the shadow cast by the planet on the ring, there where it met the Cassini division.

2. *The Rings.*

(*a*) *Outer Ring, A.*—Encke's division was seen on one occasion only, on July 30, the best night of the season (Plate 12), when it was perfectly visible on both ansæ. An easier feature of Ring A was a series of dusky indentations emerging from the Cassini division. The outer edge of A was in no wise sharply defined, but seemed to shade off rather gently into space.

Cassini's division could be traced all round the Ring without difficulty, even under very poor seeing. It was not black, but dark grey. The division seemed tangent to *Saturn's* N. limb.

(*b*) *Inner Bright Ring, B.*—This Ring shaded into the "Crape" Ring without any intermediate separation. On July 30 B was split into two rings by the certain visibility, on both ansæ, of a narrow and faint dusky line as shown in the Plate.

(*c*) *The "Crape" Ring, C,* showed nothing abnormal, excepting, perhaps, an exaggerated concavity of its inner outline towards the centre, due probably to the interference of the S. equatorial belt of *Saturn.*

The eccentricity of the Ring, noted here every year since 1895, was re-observed in 1899. The eastern vicinity was still larger than the western, though the difference was certainly less marked than a few years ago.

Juvisy Observatory (*S.-et-O.*), *France,*
1899 *December* 30.

Note on a Possible Occultation of A Geminorum by Venus, 1900 *May* 27–28. By Walter W. Bryant.

At about $2^h\ 45^m$ A.M. May 28, Greenwich civil time (some hours before the eclipse), *Venus* will be in conjunction in R.A. with *A Geminorum,* a star of the fifth magnitude, whose place for the day is approximately $7^h\ 17^m\ 24^s{\cdot}5$, N. 25° 14′ 30″. The

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SATURN

AS SEEN AT M. FLAMMARION'S OBSERVATORY ON 1899, JULY 30

BY E. M. ANTONIADI.

distance of the star from the limb of *Venus* amounts to about 9″, or half the horizontal parallax, *Venus* being south of the star.

It is practically certain that for a great part of the South Pacific the star will be occulted by *Venus* after sunset; and at some of the antipodean observatories, though there the Sun will not have set, there may be a good chance of obtaining observations of the phenomenon.

Ephemeris for Physical Observations of the Moon for the Second Half of 1900. By A. C. D. Crommelin.

Greenwich Midnight.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Geocentric Libration Lat. of the Earth.	Combined Amount.	Direction.
1900.	°	°	°	°	°	°
July 1	325·64	−0·72	+1·54	+6·74	6·91	347·1
2	337·87	−0·75	+0·34	+6·69	6·70	357·1
3	350·11	−0·77	−0·92	+6·35	6·42	8·2
4	2·33	−0·79	−2·20	+5·73	6·14	21·0
5	14·55	−0·81	−3·39	+4·87	5·93	34·8
6	26·77	−0·84	−4·45	+3·77	5·83	49·7
7	38·98	−0·86	−5·26	+2·48	5·82	64·8
8	51·18	−0·88	−5·78	+1·04	5·87	79·8
9	63·37	−0·91	−5·95	−0·48	5·97	94·6
10	75·57	−0·94	−5·72	−2·03	6·07	109·5
11	87·76	−0·96	−5·11	−3·49	6·19	124·3
12	99·95	−0·98	−4·14	−4·78	6·32	139·1
13	112·14	−1·01	−2·88	−5·78	6·46	153·5
14	124·33	−1·03	−1·45	−6·42	6·58	167·3
15	136·52	−1·05	+0·04	−6·64	6·64	180·3
16	148·73	−1·07	+1·47	−6·44	6·61	192·9
17	160·93	−1·09	+2·74	−5·82	6·43	205·2
18	173·14	−1·10	+3·78	−4·85	6·15	217·9
19	185·36	−1·12	+4·58	−3·60	5·83	231·8
20	197·59	−1·13	+5·11	−2·16	5·55	247·1
21	209·83	−1·15	+5·40	−0·63	5·42	263·3
22	222·07	−1·16	+5·46	+0·91	5·54	279·5
23	234·32	−1·18	+5·32	+2·38	5·83	294·1
24	246·57	−1·19	+4·97	+3·71	6·20	306·7
25	258·82	−1·20	+4·50	+4·83	6·60	317·0
26	271·07	−1·22	+3·82	+5·70	6·86	326·2
27	283·32	−1·23	+2·98	+6·29	6·96	334·7
28	295·57	−1·24	+1·99	+6·59	6·88	343·2

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.
1900.	°	°	°	°	°	°
July 29	307·81	−1·26	+0·85	+6·59	6·64	352·7
30	320·05	−1·27	−0·39	+6·31	6·32	3·5
31	332·29	−1·28	−1·70	+5·75	6·00	16·5
Aug. 1	344·52	−1·30	−3·01	+4·95	5·79	31·3
2	356·74	−1·31	−4·26	+3·94	5·80	47·2
3	8·96	−1·33	−5·36	+2·73	6·01	63·0
4	21·18	−1·34	−6·22	+1·37	6·36	77·6
5	33·38	−1·36	−6·76	−0·08	6·76	90·7
6	45·58	−1·37	−6·90	−1·48	7·06	102·1
7	57·78	−1·39	−6·58	−3·03	7·24	114·7
8	69·97	−1·40	−5·78	−4·35	7·23	127·0
9	82·15	−1·41	−4·53	−5·43	7·07	140·2
10	94·33	−1·42	−2·91	−6·18	6·83	154·8
11	106·51	−1·43	−1·07	−6·52	6·61	170·7
12	118·69	−1·44	+0·83	−6·40	6·45	187·4
13	130·87	−1·45	+2·62	−5·85	6·41	204·1
14	143·06	−1·45	+4·16	−4·92	6·44	220·2
15	155·26	−1·46	+5·36	−3·69	6·51	235·5
16	167·47	−1·46	+6·18	−2·26	6·58	249·9
17	179·68	−1·47	+6·62	−0·74	6·66	263·6
18	191·90	−1·47	+6·74	+0·80	6·79	276·8
19	204·13	−1·47	+6·56	+2·25	6·94	288·9
20	216·36	−1·47	+6·16	+3·56	7·12	300·0
21	228·59	−1·47	+5·56	+4·68	7·27	310·1
22	240·83	−1·47	+4·79	+5·56	7·34	319·3
23	253·07	−1·48	+3·90	+6·17	7·30	327·7
24	265·31	−1·48	+2·88	+6·49	7·10	336·1
25	277·55	−1·48	+1·76	+6·52	6·75	344·9
26	289·79	−1·48	+0·54	+6·27	6·29	355·1
27	302·02	−1·49	−0·76	+5·74	5·79	7·5
28	314·25	−1·49	−2·11	+4·97	5·40	23·0
29	326·48	−1·49	−3·45	+3·99	5·27	40·8
30	338·71	−1·49	−4·73	+2·82	5·51	59·2
31	350·93	−1·49	−5·87	+1·52	6·06	75·5
Sept. 1	3·14	−1·50	−6·79	+0·12	6·79	89·0
2	15·35	−1·50	−7·30	−1·32	7·42	100·3
3	27·54	−1·50	−7·50	−2·74	7·98	110·1
4	39·74	−1·50	−7·32	−4·05	8·37	119·0

Greenwich Midnight. 1900.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration Sel. Long. of the Earth.	Lat.	Combined Amount.	Direction.
	°	°	°	°	°	°
Sept. 5	51·92	−1·50	−6·53	−5·17	8·33	128·4
6	64·10	−1·50	−5·23	−6·00	7·96	138·9
7	76·27	−1·49	−3·49	−6·45	7·33	151·6
8	88·44	−1·49	−1·45	−6·46	6·62	167·3
9	100·61	−1·48	+0·70	−6·01	6·05	186·6
10	112·77	−1·47	+2·77	−5·13	5·83	208·4
11	124·94	−1·47	+4·59	−3·91	6·03	229·6
12	137·12	−1·46	+6·02	−2·45	6·50	247·9
13	149·31	−1·45	+7·00	−0·88	7·06	262·8
14	161·49	−1·44	+7·54	+0·70	7·57	275·3
15	173·69	−1·43	+7·66	+2·19	7·97	286·0
16	185·90	−1·41	+7·43	+3·54	8·23	295·5
17	198·10	−1·40	+6·90	+4·67	8·33	304·1
18	210·32	−1·39	+6·15	+5·56	8·29	312·1
19	222·53	−1·38	+5·22	+6·18	8·09	319·8
20	234·76	−1·37	+4·16	+6·52	7·72	327·5
21	246·98	−1·36	+3·00	+6·57	7·22	335·5
22	259·20	−1·35	+1·76	+6·32	6·46	344·4
23	271·43	−1·34	+0·47	+5·82	5·84	355·4
24	283·66	−1·33	−0·86	+5·06	5·13	9·6
25	295·88	−1·32	−2·19	+4·07	4·62	28·3
26	308·10	−1·31	−3·50	+2·91	4·55	50·3
27	320 32	−1·30	−4·74	+1·61	5·01	71·2
28	332·53	−1·29	−5·85	+0·22	5·85	87·8
29	344·74	−1·28	−6·75	−1·20	6·86	100·1
30	356·94	−1·27	−7·38	−2·60	7·82	109·4
Oct. 1	9·13	−1·25	−7·74	−3·90	8·67	116·7
2	21·32	−1·24	−7·46	−5·03	9·00	124·0
3	33·50	−1·23	−6·81	−5·92	9·02	131·0
4	45·67	−1·21	−5·66	−6·46	8·59	138·8
5	57·83	−1·19	−4·06	−6·61	7·76	148·4
6	69·99	−1·18	−2·12	−6·30	6·65	161·4
7	82·14	−1·16	+0·01	−5·55	5·55	180·1
8	94·29	−1·14	+2·13	−4·39	4·88	205·9
9	106·44	−1·11	+4·06	−2·93	5·01	234·2
10	118·60	−1·09	+5·66	−1·30	5·81	257·1
11	130·76	−1·07	+6·84	+0·37	6·85	273·1
12	142·93	−1·04	+7·55	+1·97	7·80	284·6

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat.	Geocentric Libration Sel. Long.	Lat. of the Earth.	Combined Amount.	Direction.
1900.	°	°	°	°	°	°
Oct. 13	155·10	−1·02	+7·81	+3·41	8·52	293·6
14	167·28	−1·00	+7·66	+4·63	8·95	301·2
15	179·46	−0·97	+7·16	+5·59	9·08	308·0
16	191·65	−0·95	+6·38	+6·26	8·94	314·5
17	203·85	−0·93	+5·38	+6·63	8·54	320·9
18	216·05	−0·91	+4·24	+6·71	7·94	327·7
19	228·25	−0·89	+2·99	+6·50	7·15	335·3
20	240·46	−0·87	+1·69	+6·01	6·24	344·3
21	252·67	−0·85	+0·37	+5·26	5·27	356·0
22	264·89	−0·83	−0·94	+4·29	4·39	12·4
23	277·09	−0·81	−2·21	+3·12	3·82	35·3
24	289·30	−0·79	−3·40	+1·81	3·85	62·0
25	301·50	−0·77	−4·49	+0·40	4·51	84·9
26	313·70	−0·75	−5·43	−1·05	5·53	100·9
27	325·91	−0·73	−6·17	−2·47	6·65	111·8
28	338·10	−0·71	−6·68	−3·79	7·68	119·6
29	350·29	−0·68	−6·88	−4·95	8·48	125·7
30	2·47	−0·66	−6·73	−5·88	8·94	131·1
31	14·64	−0·64	−6·20	−6·50	8·98	136·4
Nov. 1	26·81	−0·61	−5·27	−6·75	8·56	142·0
2	38·96	−0·58	−3·96	−6·58	7·68	149·0
3	51·11	−0·56	−2·36	−5·98	6·43	158·5
4	63·26	−0·53	−0·55	−4·97	5·00	173·7
5	75·40	−0·50	+1·31	−3·61	3·84	199·9
6	87·53	−0·47	+3·09	−2·00	3·68	237·1
7	99·67	−0·44	+4·65	−0·27	4·66	266·7
8	111·81	−0·41	+5·88	+1·43	6·05	283·7
9	123·95	−0·37	+6·71	+3·01	7·35	294·2
10	136·10	−0·34	+7·11	+4·36	8·34	301·5
11	148·26	−0·31	+7·10	+5·44	8·94	307·5
12	160·42	−0·28	+6·72	+6·22	9·16	312·8
13	172·58	−0·25	+6·00	+6·68	8·98	318·1
14	184·76	−0·23	+5·03	+6·83	8·48	323·6
15	196·94	−0·20	+3·88	+6·68	7·72	329·9
16	209·12	−0·17	+2·60	+6·24	6·76	337·4
17	221·31	−0·15	+1·27	+5·54	5·68	347·1
18	233·49	−0·12	−0·05	+4·60	4·60	0·6
19	245·69	−0·10	−1·32	+3·46	3·70	20·9

Greenwich Midnight. 1900.	Selenographical Colong. of the Sun.	Selenographical Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Geocentric Libration Lat. of the Earth.	Combined Amount.	Direction.
	°	°	°	°	°	°
Nov. 20	257·88	−0·07	−2·47	+2·16	3·28	48·8
21	270·07	−0·05	−3·49	+0·74	3·57	78·0
22	282·27	−0·02	−4·34	−0·74	4·40	99·7
23	294·47	0·00	−4·99	−2·20	5·45	113·8
24	306·66	+0·03	−5·42	−3·57	6·49	123·4
25	318·85	+0·05	−5·62	−4·78	7·38	130·4
26	331·04	+0·08	−5·57	−5·76	8·01	136·0
27	343·21	+0·10	−5·26	−6·44	8·32	140·8
28	355·38	+0·13	−4·70	−6·77	8·24	145·2
29	7·55	+0·16	−3·89	−6·71	7·76	149·9
30	19·71	+0·19	−2·85	−6·24	6·86	155·5
Dec. 1	31·85	+0·22	−1·63	−5·37	5·61	163·1
2	44·00	+0·25	−0·28	−4·16	4·17	176·1
3	56·13	+0·28	+1·12	−2·66	2·89	202·8
4	68·26	+0·31	+2·48	−1·00	2·67	248·0
5	80·39	+0·35	+3·73	+0·72	3·80	280·9
6	92·51	+0·38	+4·76	+2·37	5·32	296·5
7	104·64	+0·41	+5·51	+3·84	6·72	304·9
8	116·77	+0·44	+5·93	+5·05	7·79	310·4
9	128·91	+0·47	+5·98	+5·96	8·44	314·9
10	141·05	+0·50	+5·68	+6·55	8·67	319·1
11	153·20	+0·53	+5·05	+6·80	8·47	323·4
12	165·35	+0·55	+4·13	+6·73	7·90	328·5
13	177·51	+0·58	+3·00	+6·37	7·04	334·8
14	189·67	+0·61	+1·72	+5·74	5·99	343·3
15	201·84	+0·63	+0·38	+4·87	4·89	355·5
16	214·01	+0·65	−0·94	+3·79	3·90	13·9
17	226·19	+0·68	−2·17	+2·54	3·35	40·5
18	238·37	+0·70	−3·24	+1·15	3·44	70·5
19	250·56	+0·72	−4·09	−0·30	4·10	94·2
20	262·74	+0·74	−4·68	−1·78	5·01	110·8
21	274·93	+0·76	−4·99	−3·18	5·92	122·5
22	287·12	+0·78	−5·01	−4·45	6·70	131·6
23	299·31	+0·80	−4·77	−5·50	7·28	139·1
24	311·49	+0·83	−4·28	−6·25	7·57	145·6
25	323·68	+0·85	−3·61	−6·65	7·57	151·5
26	335·85	+0·87	−2·79	−6·66	7·22	157·3
27	348·02	+0·90	−1·88	−6·27	6·55	163·3

Greenwich Midnight.	Selenographical Colong. of the Sun.	Lat. of the Sun.	Geocentric Libration Sel. Long. of the Earth.	Lat. of the Earth.	Combined Amount.	Direction.
1900.	°	°	°	°	°	°
Dec. 28	0·18	+0·92	−0·92	−5·50	5·58	170·5
29	12·34	+0·94	+0·05	−4·39	4·39	180·7
30	24·48	+0·97	+1·02	−3·01	3·18	198·7
31	36·62	+0·99	+1·95	−1·45	2·43	233·4
1901.						
Jan. 1	48·76	+1·02	+2·82	+0·19	2·83	273·9

The longitudes are reckoned in the plane of the Moon's equator, the axis of reference being the radius which passes through the mean centre of the visible disc. This axis therefore rotates with the Moon, and is not fixed in space.

The inclination of the Moon's equator to the ecliptic is taken as 1°·523, the value used in the *Connaissance des Temps*, that given by the *Nautical Almanac* being 1°·536.

The physical librations in longitude and latitude, as given by Professor Franz's formulæ, have been applied; their values are taken from the *Berliner Jahrbuch* for the days given there, and interpolated by a graphical method for the other days. But the signs in the *Jahrbuch* require to be reversed in order to reduce to the system used here.

The colongitude of the Sun is 90° (or 450°) *minus* his selenographical longitude. It also is the selenographical longitude of the morning terminator reckoned eastward from the mean centre of the disc. Hence its value is approximately 270°, 0°, 90°, 180° at new Moon, first quarter, full Moon, last quarter respectively. The longitude of the evening terminator is of course 180° greater or less than that of the morning one.

When the geocentric libration in longitude is positive, the region brought into view is on the west limb; when negative, on the east.

When the geocentric libration in latitude is positive, the region brought into view is at the Moon's north pole; when negative, at the south.

The column "Combined Amount" gives the distance between the apparent and mean centres of the disc, and the column "Direction" gives the position-angle of the apparent centre from the mean centre, or, which is the same thing, the position-angle of the region which is most carried into view by libration. The angles are reckoned eastward from the northern extremity of the Moon's axis.

The terms "East" and "West" are used throughout with reference to our sky, and not as they would appear to an observer on the Moon.

Benvenue, 55 *Ulundi Road, Blackheath, S.E.*
1900 *February* 13.

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